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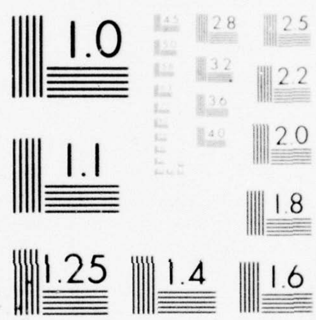

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CONVEX CHUTES IN CONVERGING SUPERCRITICAL FLOW

by

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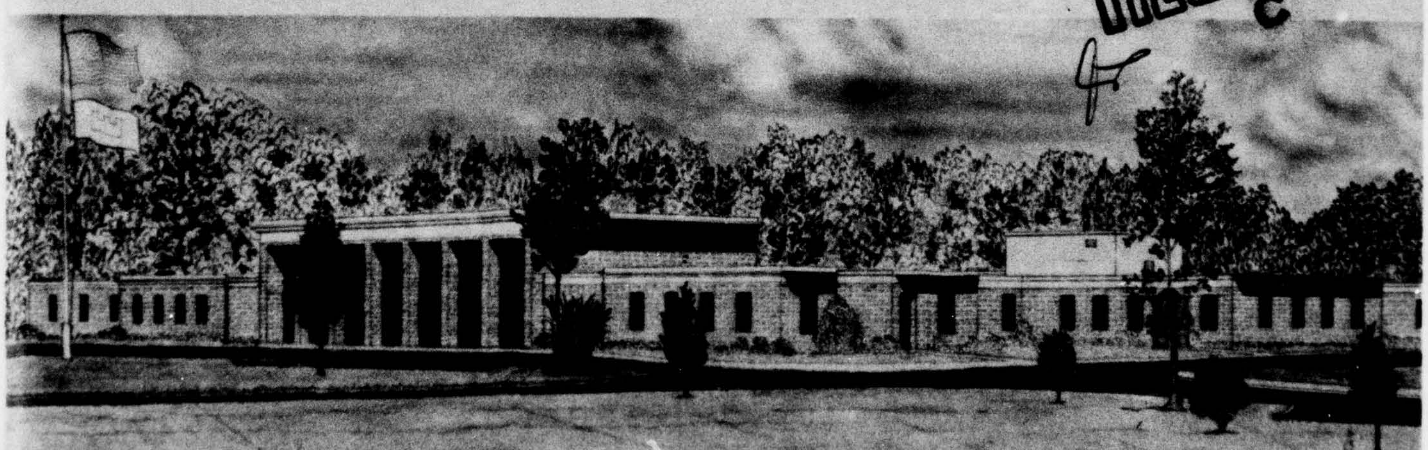

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This study contains a brief literature survey and limited experimental data regarding the water-surface contours for supercritical flow in a steep converging chute. Items relevant to design of the chute sidewalls are described, and the development of a computer model for chute design is suggested. The author also suggests that a horizontal transverse water surface cannot be obtained unless parallel sidewalls are used and that for a converging chute, the designer will probably have to evaluate and accept some level of nonuniformity in the flow.

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PREFACE

This study was conducted during the period October 1972 through June 1976 by personnel of the U. S. Army Engineer Waterways Experiment Station (WES) and in the In-House Laboratory Independent Research (ILIR) programs of the Corps of Engineers. Dr. G. G. Quarles, Office of Chief of Engineers, was the program monitor.

The work was conducted under the supervision of Messrs. H. B. Simmons, Chief of the Hydraulics Laboratory, and E. B. Pickett, Chief of the Hydraulic Analysis Division. Dr. F. M. Neilson directed the work and prepared this report. Acknowledgment is made to Mr. J. E. Hall for his effort in constructing the test facility and conducting the tests reported herein.

Directors of WES during the conduct of the study and the preparation and publication of this report were COL G. H. Hilt, CE, and COL John L. Cannon, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
inches	25.4	millimetres
feet	0.3048	metres
square feet	0.09290304	square metres
cubic feet per second	0.02831685	cubic metres per second
feet per second per second	0.3048	metres per second per second
degrees (angular)	0.01745329	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins*

* To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: $C = (5/9)(F - 32)$. To obtain Kelvin (K) readings, use: $K = (5/9)(F - 32) = 273.15$.

CONVEX BOUNDARIES IN CONVERGING SUPERCRITICAL FLOW

PART I: INTRODUCTION

Statement of the Problem

1. Steep chutes are commonly used in hydraulic structures as conveyances for high-velocity flows between a spillway crest and an energy dissipator. Normally, the type, width, and elevation of the crest are determined by the stage-discharge relationship prescribed by overall project requirements. Similarly, the width and elevation of the floor at the entrance to the energy dissipator (often a stilling basin) are determined by the need to match the hydraulic characteristics of the dissipator with the stage-discharge relationship prescribed by downstream channel requirements. Whenever the crest length is greater than the width of the energy dissipator, the chute must converge in the downstream direction. The subject of this study is the shape in plan of the chute sidewalls; the object is to study the relationships commonly used to design an acceptable rate of curvature of the sidewalls. The rate of curvature is a function of the hydraulic characteristics of the flow as well as of the geometrical characteristics imposed on the chute by general project requirements.

Practical Application

2. The project features that most commonly enter into the overall chute spillway design are the approach channel, wing walls, spillway crest, chute, and energy dissipator (Figures 1a and 1b). In hydraulic engineering manuals and in research studies, these items are often treated separately; in the overall project, however, the designs must be integrated so that acceptable flow conditions exist throughout. Hydraulic model tests are the most common design tool used to select the best overall geometry in situations not previously encountered and not

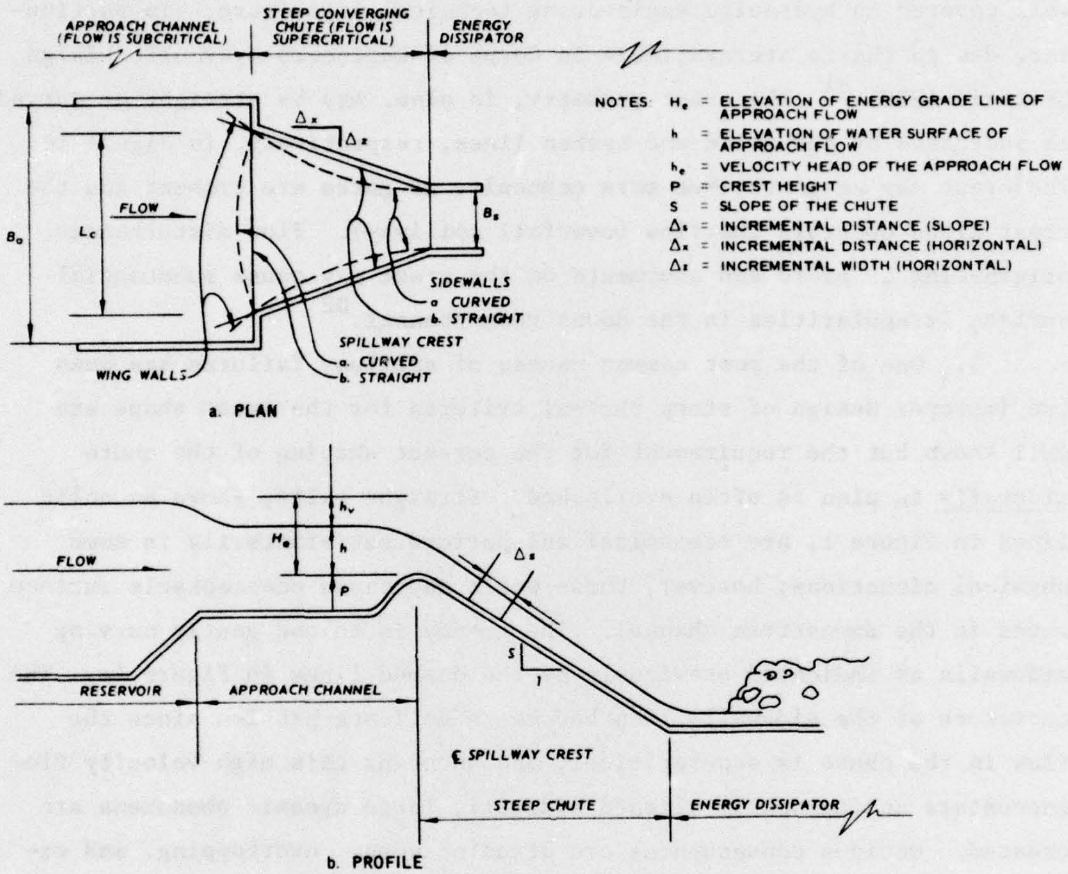


Figure 1. Typical steep chute

amenable to analytical solution. A few general considerations that influenced the approach taken in this study are outlined in the following paragraphs for each of the above project features.

3. The upstream approach to the spillway should align the flow normal to the crest--an improperly designed approach can impose flow disturbances that are amplified along the chute and are unacceptable in the downstream channel.[D1] The wing walls usually should extend upstream from the crest so that the initial contraction occurs in a subcritical flow regime; they may be curved in plan so as to prevent vortex shedding at the upstream edge.

4. The design procedure for the profile of the spillway crest is well covered in hydraulic engineering technical literature; in particular, design charts are available in Corps of Engineers Hydraulic Design Criteria (HDC).[A1] The crest geometry, in plan, may be straight or curved as indicated by the solid and broken lines, respectively, in Figure 1a. The crest may be gated; but more commonly, no gates are present and the crest alone controls the flow (overfall spillway). Flow disturbances originating at piers and abutments on the crest may cause substantial surface irregularities in the downstream channel.[D2]

5. One of the most common causes of spillway failures has been the improper design of steep chutes; criteria for the crest shape are well known but the requirement for the correct shaping of the chute sidewalls in plan is often overlooked. Straight walls, shown as solid lines in Figure 1, are economical and perform satisfactorily in some physical situations; however, these walls may cause unacceptable surface waves in the downstream channel. The remedy is to use gently curving sidewalls as indicated previously by the dashed lines in Figure 1a. The curvature of the sidewalls then becomes a delicate problem since the flow in the chute is supercritical, and whenever this high-velocity flow encounters an improperly aligned sidewall, large dynamic phenomena are created. Obvious consequences are standing waves, overtopping, and excessive structural loadings as well as a nonuniform flow in the downstream channel.

6. Energy dissipation below the chute is normally accomplished by means of plunge pools, deflector buckets, or stilling basins. For plunge pools and deflector buckets some degree of nonuniformity may be acceptable in the flow as it leaves the chute, depending on the physical characteristics of the exit channel. However, in the case of a stilling basin, the flow should be as nearly uniform as possible.

Review of the Literature

7. Converging chutes have been used in numerous projects and often successfully. The high percentage of successes is due to the major role that physical hydraulic models play in a comprehensive hydraulics design program. Grand Coulee Dam,[D3] for example, which uses a parallel-walled chute, required 14 separate models representing designs from the planning to the construction phase in order to obtain the final adopted configuration. Surface shock waves in the flow in the chute are easily observed in a reasonably sized spillway model. Consequently, the configuration of the chute (or appurtenant structural components) can be modified in the model to eliminate (or reduce to acceptable levels) the amplitude of the waves. Of course, in order to reduce the effort in the laboratory, a good initial design is a tremendous advantage.

8. Common configurations of chute spillways and the associated design criteria, as of 31 March 1965, of the Corps of Engineers are given in Engineer Manual (EM) 1110-2-1603[A2]; comparable information for flood control channels as of 1 July 1970 is contained in EM 1110-2-1601.[A3] The present suggested criteria, including data from nine existing chutes, were reviewed by R. G. Cox[C1] and published as Engineer Technical Letter (ETL) 1110-2-158[A4] during 1972. Design criteria have also been described for the Bureau of Reclamation[A5] and in numerous hydraulic engineering treatises (References A6, A7, and A8, for example). An overview of published design guidance for situations in which standing waves are to be avoided and uniform flow is required at the entrance to the downstream energy dissipator is as follows.

- a. Optimum flow conditions[A4] occur with a crest formed by a break in invert grade or by a low sill formed as an integral part of the chute slope; this precludes ogee crests with toe curves, although this type of design has been used successfully in practice. Curving the chute crest does not appreciably affect the flow conditions in the chute.
- b. Straight, parallel sidewalls are preferred whereas reverse curves are normally unacceptable; where required, gradually convergent or divergent walls can be used.

c. Vertical walls are preferred[A4]; if sloping walls are used, the side slope should be considered relative to the chute invert rather than to the horizontal plane.

d. The walls should be straight in the vicinity of the crest; the convergence ratio at the crest, $|\Delta s/\Delta z|$, should be greater than or equal to 5.0. The incremental distance along the chute, Δs, and the incremental width, Δz, are shown in Figure 1. The straight walls should extend downstream beyond the point where the local Froude number becomes greater than 1.5.[A4] The local Froude number,

$$F = \frac{v}{\sqrt{gd}}$$

is based on the local velocity, v, and local depth, d.

e. Local Froude numbers greater than 3.0 should be avoided[A4]; the Froude number should be gradually increasing throughout the transition.

f. The maximum permissible degree of curvature of the sidewalls for either convergent or divergent conditions is[A5]

$$|\Delta z/\Delta s| = \tan^{-1}\left(\frac{1}{3\bar{F}}\right) \qquad (1)$$

where $\bar{F} = \frac{\bar{v}}{\sqrt{g\bar{d}}}$; $\bar{v}$ and $\bar{d}$ are the averages of velocities and depths at the beginning and at the end of the transition.

g. A convergence should be such that[A4]

$$\frac{\Sigma\Delta s}{\Sigma\Delta z} = \frac{1}{0.382 - 0.116F} \qquad (2)$$

in which $\Sigma\Delta s$ is the center-line station distance measured along the slope downstream from the center line of the crest, $\Sigma\Delta z$ is the horizontal convergence of one sidewall, and F is the local Froude number.

9. Surface waves in supercritical flow are analogous to shock waves in compressible gases.[F1, F2] This analogy has permitted a reciprocity of information between studies concerned with liquid flows and those with gas flows. Two early works that demonstrate this information exchange are: a study by Preiswerk,[F3] who applied the methods of gas dynamics to free surface water flows; and a study by Laitone,[F4] who used

liquid flows in a study of transonic gas dynamics. A summary of the equations which describe the analogy as outlined by Crossley[F4] is as follows:

Liquid Flow	Gas Dynamics
$\frac{1}{h}\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right) = \text{constant}$	$T^{\frac{-1}{\gamma-1}}\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right) = \text{constant}$
$\frac{\partial}{\partial x}(hu_x) + \frac{\partial}{\partial y}(hu_y) = 0$	$\frac{\partial}{\partial x}\left(u_x T^{\frac{1}{\gamma-1}}\right) + \frac{\partial}{\partial y}\left(u_y T^{\frac{1}{\gamma-1}}\right) = 0$
$gh + \frac{1}{2}(u_x^2 + u_y^2) = \text{constant}$	$C_p T + \frac{1}{2}(u_x^2 + u_y^2) = \text{constant}$

in which: h = water depth

u_x, u_y = velocities in the x and y directions, respectively

g = acceleration due to gravity

T = absolute temperature

γ = ratio of specific heats

C_p = specific heat at constant pressure

The analogous quantities, given an ideal gas having $\gamma = 2$, are gh to C_pT, h/h_o to T/T_o, h/h_o to ρ/ρ_o, and h^2/h_o^2 to p/p_o. Of course, the analogy is subject to numerous and serious limitations (for example, the significance of energy dissipation as well as the physical characteristics of the gas, $\gamma \neq 2$, must be taken into account). For more detailed information on the validity of this analogy, the reader is referred to Crossley[F2] and to Harleman.[F1]

10. The analysis which is the current basis for the design of transitions for supercritical flow is summarized in a series of publications presented to the ASCE Symposium on High-Velocity Flow in Open Channels in 1948. In contrast to the empirical observations relevant to

specific design problems (some are listed in paragraph 8), these studies capitalized on the insight obtained through the supercritical-supersonic analogy described above to obtain the basic physical principles for designing transitions for supercritical flow.

11. Ippen[E1] describes the mechanics of supercritical flow for converging boundaries. The practical application pertains to open-channel flow over a horizontal floor in which surface disturbances appear as a consequence of the geometry of the lateral boundaries. Two distinct methods of approach are presented:

- a. Gradual surface changes are analyzed on the basis of constant specific head.
- b. Standing wave fronts of appreciable height (oblique or slanting hydraulic jumps) are analyzed with a consideration of the energy dissipation involved.

The following shock-wave intersections and reflections are included:

- a. Reflection of shock wave at opposite wall.
- b. Intersection of shock waves of equal intensity and of unequal intensity.
- c. Convergence of two shock fronts.
- d. Shock front and negative deflection waves.

12. Ippen and Dawson[E2] apply the mechanics of supercritical flow directly to the design of channel contractions. Examples of the computational procedure are presented, and comparisons with experimental results are made. Of particular interest, experimental water-surface elevation contours for subcritical ($F<1$) and supercritical ($F>1$) flows through two contractions are included.

13. High-velocity flow in open-channel expansions is analyzed by Rouse, Bhoota, and Hsu[E3]; in general, water-surface contours were determined analytically (by means of the method of characteristics) for numerous experimental transition designs which were then tested in the laboratory. The experimental verification is reasonable at mild slopes, less so at steep slopes (10 percent is the steepest slope tested). For mild slopes, the data are generalized (not a rigorous identity) in the form of

$$\frac{d}{d_1} = f\left(\frac{s}{B_1F_1}, \frac{z}{B_1}\right) \qquad (3)$$

in which s and z are the longitudinal and transverse coordinates, respectively; B is the channel width; and the subscript 1 refers to conditions at the entrance to the expansion.

14. The basic formulation and design procedures, as presented by Ippen[E1] have recently been incorporated into a computer spillway design procedure by Taubert.[C2] The computer program is suited to funnel-shaped (straight crest in Figure 1) and to nozzle-shaped (which has a reverse curve in the sidewalls) contractions; the program is not applicable to fan-shaped contractions (curved crest in Figure 1). The program consists of five basic algorithms as follows:

- a. Algorithm 1 is used to calculate the characteristics of standing wave fronts caused by small deflections (specific head is constant).
- b. Algorithm 2 is used to calculate the characteristics of standing waves with energy dissipation (oblique shocks).
- c. Algorithm 3 is concerned with wave reflection from a vertical wall. The angle of deflection at the reflected wave front is set equal to that of the arriving front; algorithm 1 or 2 (depending on energy dissipation considerations) is used to advance across the new wave.
- d. Algorithm 4 is concerned with the intersection of two waves, either of equal or of unequal intensity.
- e. Algorithm 5 is concerned with the convergence of two wave fronts.

15. Taubert[C2] points out that the basic equations assume a horizontal floor and uniform flow and that these assumptions are not valid in practice. These two effects tend to compensate for one another. However, when the influence of the floor slope is strongly dominant, an elongation of the flow event occurs.[E3] The effect of vertical acceleration at the crest of a chute, which also was not included in the basic formulation, is described by Gunzel.[C3]

16. Most flow phenomena occurring in steep chutes with straight parallel walls are also of concern in the design of converging chutes. Some observations and studies that are of particular concern are as follows:

a. The flow near the crest is highly nonuniform; however, the upper nappe profile in this region may be approximated by means of successive potential flow (flow net) solutions. The uppermost streamline is adjusted between solutions and is finally located such that the total head is constant along the free surface (see Henderson[A8] and Cassidy,[C4,C5] for example). Gunzel[C3] presents experimental data concerning the effects of the rapidly varied flow near the crest of spillways. Cox[B1] presents empirical curves which can be used to locate the upper nappe profile for flows over crests of particular shapes. The slope of the free surface at control points is the subject of a recent study by Wilkinson.[B2]

b. The gradually varied profile farther downstream from the crest is normally an S2 curve which may be calculated by standard methods (see Chow,[A6] for example). The USBR[A5] design guide contains a nomograph that simplifies these backwater computations for a steep chute.

c. Liggett and Vasudev[B3] discuss slope and frictional effects in two-dimensional supercritical flow. A Corps of Engineers publication[B4] describes the development of the turbulent boundary layer (and a means for evaluating the corresponding energy losses) downstream from a spillway crest.

d. Air entrainment must be considered for flows in long chutes. This phenomenon often occurs in prototype structures; however, it is a rare occurrence in hydraulic models. A recent study by Keller, et al.,[B5] discusses the air entrainment phenomenon and presents a means for determining the point along the chute at which self-aeration of the flow begins.

e. Flow instability can occur in a steep chute; the resulting unsteady flow is termed roll waves or slug flow. Roll waves are discussed in numerous publications[G1,G2,G3]; the presentation by Mayer[G3] and the discussions thereof constitute a reasonably complete background of the subject.

Current Study

17. Each aspect of the overall spillway design problem noted previously in the brief literature review has been studied, and in many regards resolved, by earlier investigations. The study herein is

therefore reduced to the following items.

a. Construction and operation of a physical-model test facility to provide information regarding:

   (1) The point of origin along the wing wall at which the first standing wave occurs (convergence).

   (2) The geometry of the first wave front (convergence).

   (3) The geometry of the flow for the subsequent diverging-wall situation (divergence).

b. Suggesting steps to be taken in order to design a satisfactory shape in plan of a chute sidewall.

PART II: EXPERIMENTAL APPARATUS

Description

18. Figure 2 shows the experimental facility; details of the apparatus are presented in Plate 1.

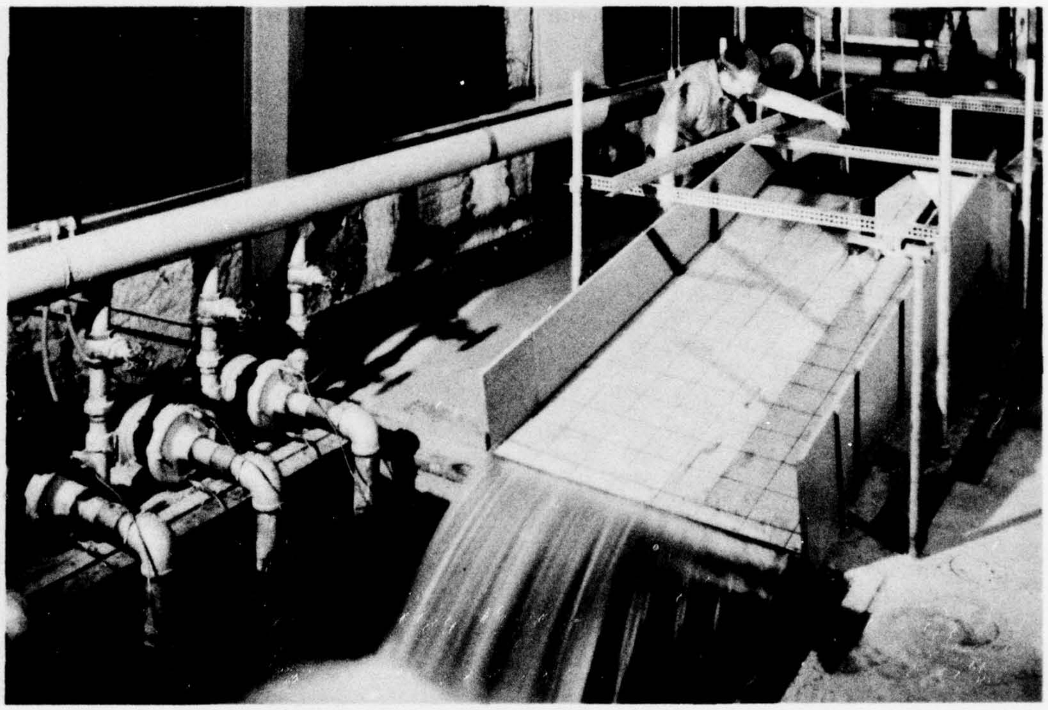


Figure 2. Experimental facility

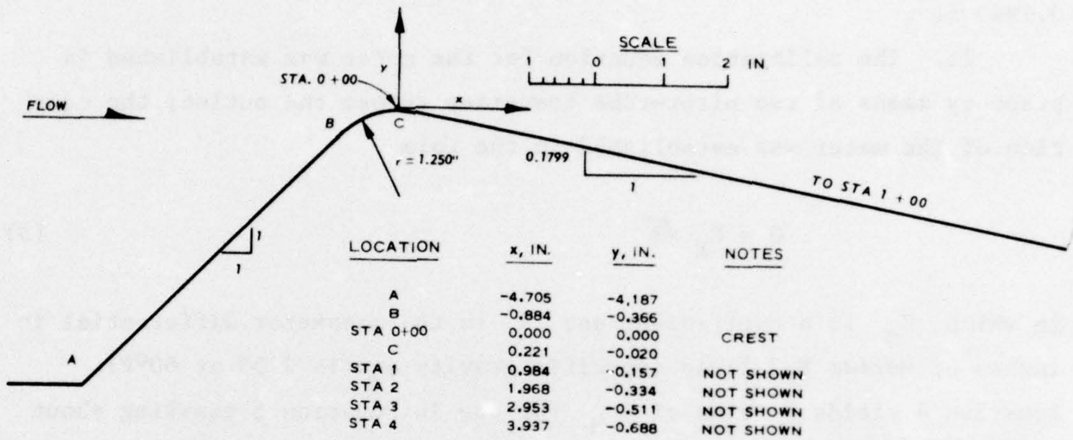
A summary of the basic components follows:

a. The supply system, from the sump at the toe of the chute to the headbay, contained a pumping unit made up of four 1-hp centrifugal pumps in parallel that discharged through a 6-in.-diam delivery line. The maximum capacity of the system was about 2.25 cfs; flow control was by means of a 6-in. gate valve in the delivery line.

b. The dimensions of the headbay and approach channel are shown in Plate 1. The geometry is such that the flow lines approaching the crest are parallel in plan. Horsehair and screen matting were used as baffles in the headbay so that surface irregularities were essentially

eliminated before reaching the approach channel.

c. The dimensions of the chute spillway are also shown in Plate 1. Although the slope of the chute was variable, a specific slope of 0.1799 on 1 (equaling 10.2 degrees) was used in all tests reported herein. Details of the profile of the spillway crest are shown in Figure 3.



LOCATION	x, IN.	y, IN.	NOTES
A	-4.705	-4.187	
B	-0.884	-0.366	
STA 0+00	0.000	0.000	CREST
C	0.221	-0.020	
STA 1	0.984	-0.157	NOT SHOWN
STA 2	1.968	-0.334	NOT SHOWN
STA 3	2.953	-0.511	NOT SHOWN
STA 4	3.937	-0.688	NOT SHOWN

Figure 3. Details of the spillway crest

d. The basic measuring equipment consisted of a 90° bend meter for discharge and point gages for water-surface elevation. Dye and confetti were used on occasion for visual observations of flow patterns.

General Hydraulic Parameters

19. The three basic coefficients that typify the apparatus involved (a) calibration of the bend meter, (b) determination of a free-flow crest discharge coefficient, and (c) evaluation of the hydraulic roughness of the steep chute. These items are discussed in the following paragraphs.

20. An approximate relationship for discharge measurement by means of a 90° bend meter is

$$Q = \sqrt{\frac{r}{2D}}\; a\sqrt{2g\Delta h} \qquad (4)$$

in which r is the center-line radius of the pipe bend, D and a are respectively the pipe diameter and cross-sectional area, g is the gravitational acceleration, and h is the difference in piezometric head between the inside and outside of the bend. For the experimental setup shown in Plate 1, r was 0.667 ft, D was 0.5 ft, and a was 0.1963 ft^2.

21. The calibration equation for the meter was established in place by means of two pitot-tube traverses across the outlet; the equation of the meter was established in the form

$$Q = C_M \sqrt{R} \tag{5}$$

in which C_M is a coefficient and R is the manometer differential in inches of Meriam M-3 fluid (specific gravity equals 2.95 at 60°F). Equation 4 yields a value of C_M for use in Equation 5 equaling about 0.52; the in place average calibration value, C_M equals 0.55, was used in all data reduction herein and should be accurate within ±5 percent over the range of discharges of interest. Both curves are shown in Figure 4; the Reynolds* number, R_e, ranged from $1.4(10^5)$ to $4.9(10^5)$ for the discharges used in the test program.

22. The crest discharge coefficient, C, was evaluated from the following equation[A1]

$$Q = C\,L'\,H_e^{3/2} \tag{6}$$

in which L' is the effective crest length and H_e, which is the elevation of the energy grade line of the approach flow relative to the crest, equals $h + h_v$ as previously shown in Figure 1. For a crest with one rectangular abutment[A1]

$$L' = L_c - 0.1\,H_e \tag{7}$$

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* $R_e = \frac{VD}{\nu}$ in which $V = Q/a$ is the mean velocity in the delivery line and ν is the kinematic viscosity.

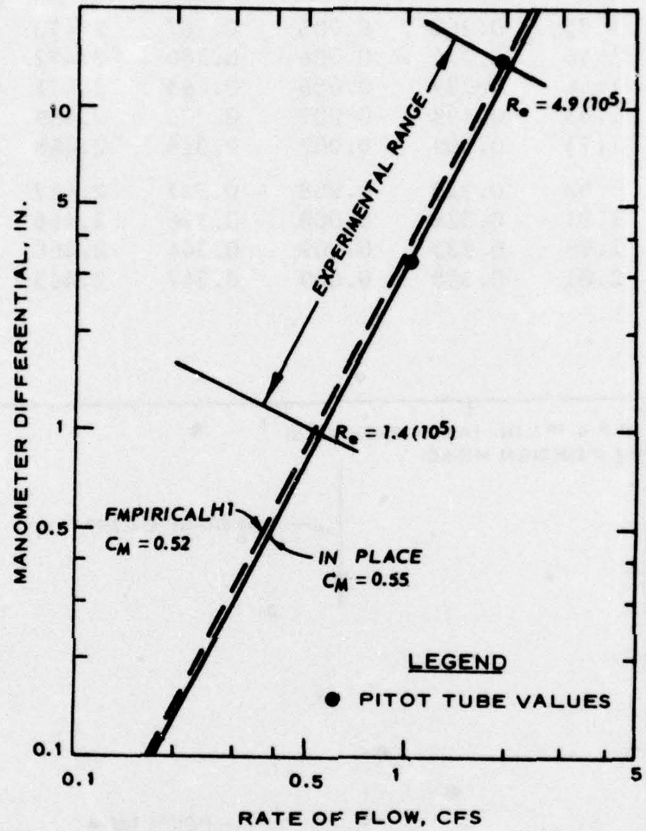


Figure 4. Bend meter calibration

in which L_c is the crest length (2.500 ft for the test facility). A series of observations, tests H1-H14, were performed in order to determine C as a function of H_e; the data and calculations are tabulated below, and the results are summarized in Figure 5.

Test No.	Q cfs	h ft	h_v ft	H_e ft	L' ft	C
H1	0.55	0.143	0.001	0.144	2.486	4.05
H2	0.78	0.181	0.002	0.183	2.482	4.01
H3	0.95	0.207	0.003	0.210	2.479	3.98
H4	1.10	0.229	0.004	0.233	2.477	3.95
H5	1.23	0.248	0.005	0.253	2.475	3.91

(Continued)

Test No.	Q cfs	h ft	h_v ft	H_e ft	L' ft	C
H6	1.35	0.262	0.005	0.267	2.473	3.96
H7	1.46	0.274	0.006	0.280	2.472	3.99
H8	1.56	0.289	0.006	0.295	2.471	3.94
H9	1.65	0.299	0.007	0.306	2.469	3.95
H10	1.74	0.309	0.007	0.316	2.468	3.97
H11	1.82	0.319	0.008	0.327	2.467	3.95
H12	1.91	0.328	0.008	0.336	2.466	3.98
H13	1.98	0.335	0.009	0.344	2.466	3.98
H14	2.01	0.338	0.009	0.347	2.465	3.99

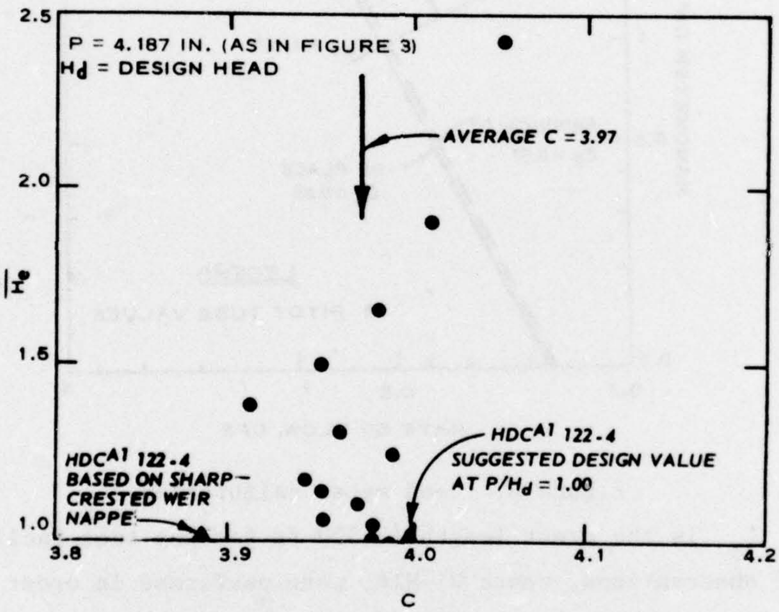


Figure 5. Crest discharge coefficient (parallel sidewalls)

23. The average value, C = 3.97, is used herein. The largest difference between an observed value and the mean is 2 percent, which is substantially less than the ±5 percent confidence in the discharge measurement.

24. A sketch of the flow profile downstream from the crest is presented in Figure 6. The two parameters of interest are the ratio of depth at the crest to critical depth, $d_{0.00}/d_c$, and the evaluation of

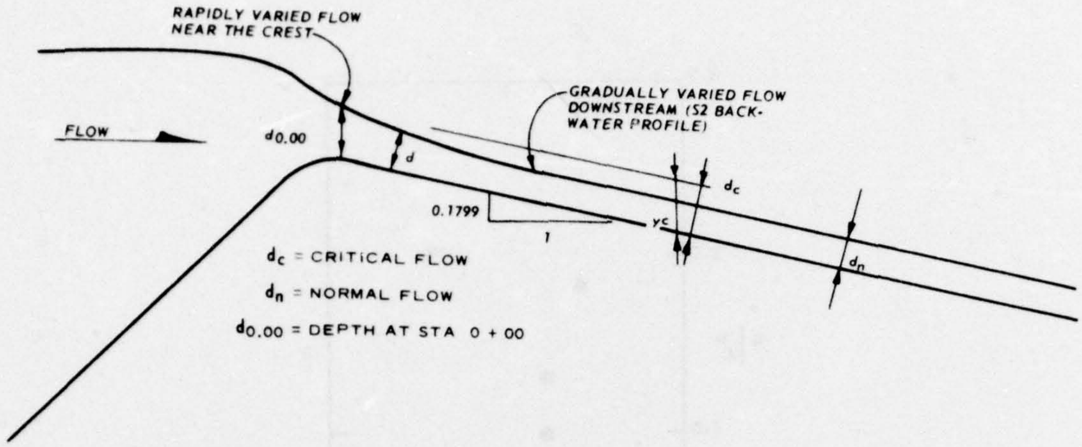


Figure 6. General flow profile (parallel sidewalls)

the normal depth, d_n, as a function of unit discharge, q. Data regarding the depth at the crest are tabulated below and shown in Figure 7; the critical depth is evaluated using the equation[A9]

$$d_c = \sqrt[3]{\frac{q^2}{g}} \tag{8}$$

Test No.	q cfs/ft	d_c ft	$d_{0.00}$ ft	$\frac{d_{0.00}}{d_c}$
H1	0.221	0.115	0.108	0.939
H2	0.314	0.145	0.139	0.958
H3	0.383	0.166	0.163	0.978
H4	0.444	0.183	0.179	0.979
H5	0.497	0.197	0.194	0.985
H6	0.546	0.210	0.209	0.996
H7	0.591	0.221	0.219	0.991
H8	0.631	0.231	0.232	1.001
H9	0.668	0.240	0.242	1.006
H10	0.705	0.249	0.248	0.994
H11	0.738	0.257	0.256	0.997
H12	0.775	0.265	0.275	1.000
H13	0.803	0.272	0.271	0.997
H14	0.815	0.274	0.277	1.008

25. **Five measured water-surface profiles (tests NH1-NH5) are shown in Figure 8. Because of the limited length of the chute, the flow became essentially uniform only for small discharges. For example,**

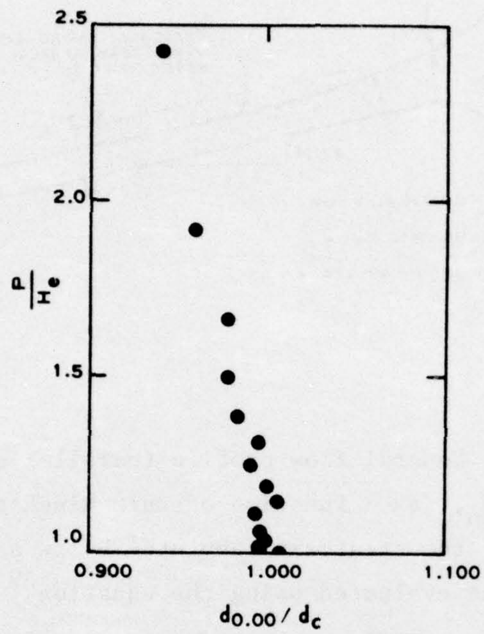


Figure 7. Depth at the spillway crest (parallel sidewalls)

in Figure 8, only the lowest flow rate, 0.319 cfs/ft, appears to be near a uniform flow condition at the extreme measurement location, sta 2+0.50 ft. This test condition was used to evaluate a roughness coefficient, n, for Manning's flow equation

$$n = \frac{1.486\, y_n^{5/3}\, S_f^{1/2}}{q} \tag{9}$$

With normal depth $d_n = 0.049$ ft, $S_f = S_o = 0.1771$, and $q = 0.319$ cfs/ft in Equation 9, a value of $n = 0.0124$ was obtained. The value $n = 0.012$, which agrees with published values[A6] for a planed wood surface, was used in subsequent computations herein.

26. S2 backwater profiles for the five flows are also shown in Figure 8; these profiles are computed numerically in the form

$$y_{i+1} = y_i + \Delta y \tag{10a}$$

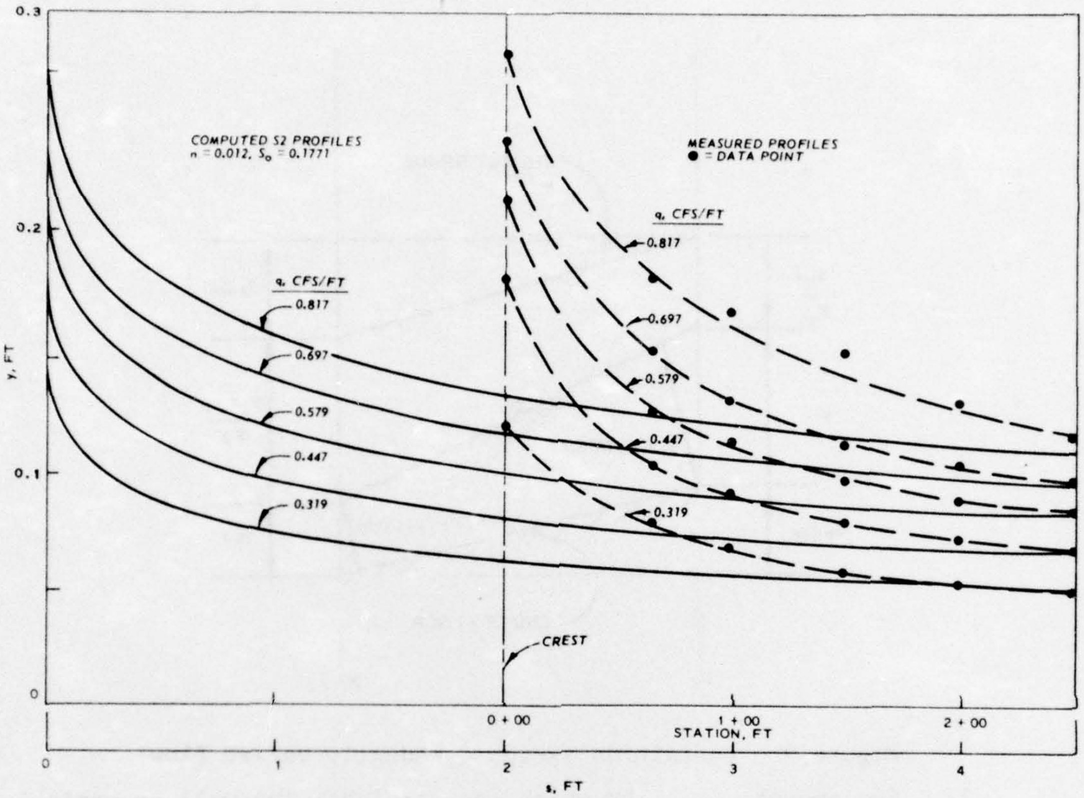


Figure 8. S2 backwater profiles (parallel sidewalls)

$$x_{i+1} = x_i + \frac{\frac{1}{2g}\left(v_{i+1}^2 - v_i^2\right) + y_{i+1} - y_i}{S_o - S_f} \tag{10b}$$

A definition sketch is shown in Figure 9. In performing the numerical integrations y is held constant, $y_i = y_c$, $x_i = 0.0$, and S_f is evaluated using Manning's equation. In plotting Figure 8, the slope coordinate, s, was used rather than the horizontal coordinate, x, and the backwater profiles are arbitrarily offset 2.00 ft along the s-axis. Because of the rapidly varying flow conditions near the crest the S2-profiles and the measured profiles are dissimilar except at larger s values; consequently, the computational technique,[A5] wherein the offset at the crest is used to obtain closer agreement between calculated and actual water-surface profiles, is not well suited to the profile near the crest.

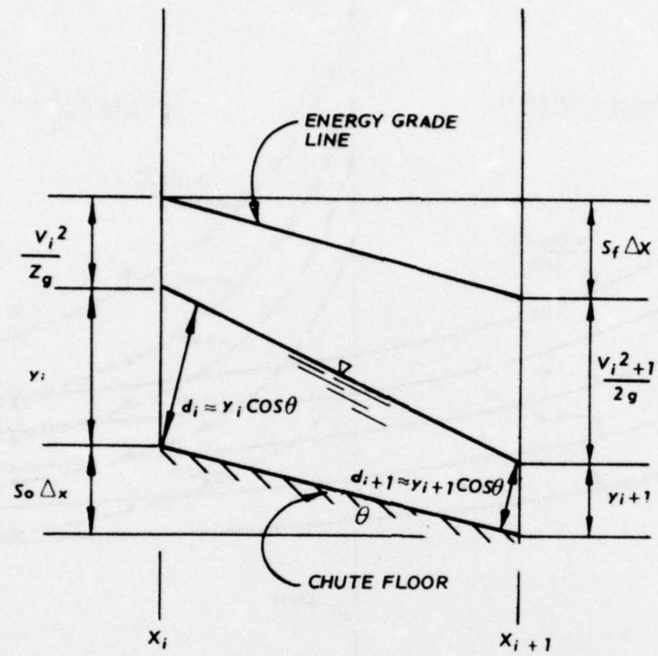
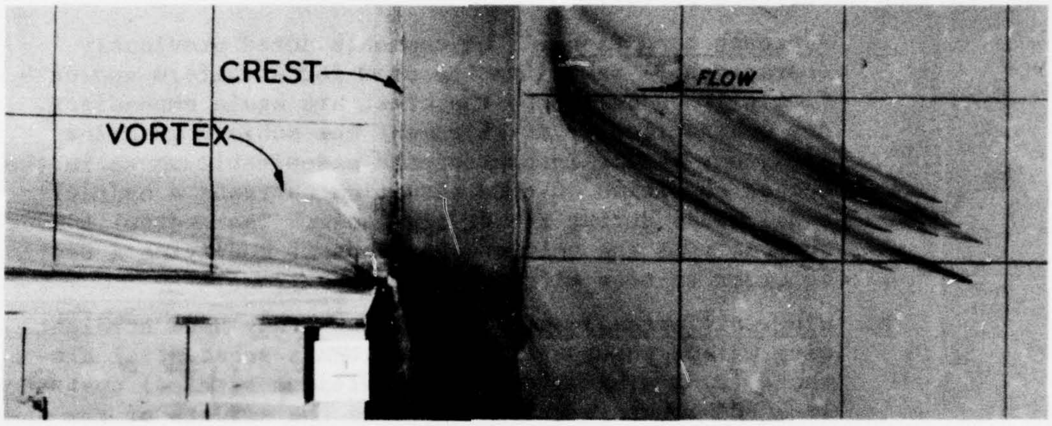
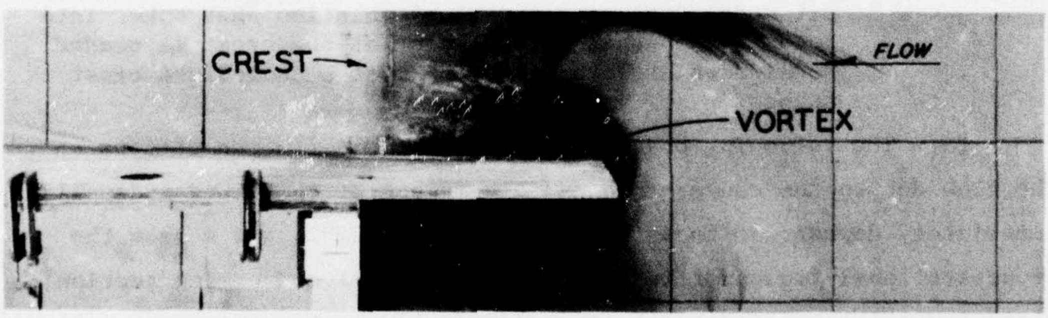


Figure 9. Definition sketch (gradually varied flow)

27. The amount, L_w, by which the straight wing-wall segments must be extended upstream in order to obtain parallel flow at the crest was determined by a separate series of tests. Eight wing-wall segments, L_w ranging from 0.33 to 3.0 ft, were used. The discharge was gradually increased until the nonparallel streamlines (as observed by means of dye streaks in the flow) originating from the vortex at the leading edge of the walls persisted to the crest. The two photographs in Figure 10 show the vortex with two wing-wall conditions at a low discharge value. The results indicated that the minimum acceptable value for the complete range of discharges studied in the test chute was 1.33 ft or L_w/p equals 3.8; this extension was used for all subsequent tests.



a. No extension



b. Extension equals 0.67 ft

Figure 10. Wing-wall extensions (Q = 0.6 cfs)

PART III: TEST OBSERVATIONS AND RESULTS

28. The following observations are presented on the basis of preliminary visual inspection of various flows in the chute coupled with the previous literature survey.

a. Approach conditions. The comments noted previously (paragraph 3) regarding the need for a uniform approach flow aligned normal to the crest are again emphasized. Any obstruction or misalignment not subject to prior study may result in unpredicted undesirable waves in the chute; suspicion of this situation warrants a hydraulic model study during the design phase. The hydraulic model currently is the only proven means of modifying a design in order to cope with poor approach flows.

b. Wing-wall extensions. The current study used straight wing walls; these walls must extend a substantial distance (3.8p minimum for the conditions studied) upstream from the crest in order to reduce the effects of the upstream vortices. In practice, these walls are usually curved in plan, thereby reducing the upstream vortices and making shorter wing walls feasible.

c. Rapidly varied flow at the crest. The gradually varied S2 flow profile does not approximate the flow conditions immediately downstream from the crest even when parallel sidewalls are used (see Figure 9). Hence, either hydraulic model data or a numerical solution that takes into account the rapid acceleration at the crest is needed. Model data are available for most standardized crest shapes.

29. The following observations have to do with the effects on the flow due to the convergence and divergence of the chute sidewalls immediately downstream from the crest. Plates 2, 3, and 4 show the cross-sectional flow profiles for a straight wall converging section; Plate 5 shows a straight wall converging followed by a straight wall diverging section. The s and z coordinate system is also shown in Plate 5. The data indicate the following.

a. Flow profile (cross section) at the crest. With straight approach flow, a two-dimensional flow situation exists and the depth at the crest is approximately equal to critical depth as previously shown in Figure 8. On the other hand, with a converging wing wall, the flow is three-dimensional, for which the depth at the crest increases nearer to the wall. Evaluating critical depth,

d_c , as $d_c = \left[(3.97)^2/g\right]^{1/3} H_e$, the observed change is as follows:

Test	Q cfs	p/H_e	$d_{0.000}/d_c$ Near Wing Wall	$z = 0.0$
N1	0.590	2.29	1.13	0.99
N2	0.992	1.62	1.09	0.98
N3	1.481	1.22	1.05	0.96

b. Type of standing wave (convergence). If the design head, H_D, for the crest is assumed to be twice the radius of the crest, then the values of H_e/H_D for tests N1, N2, and N3 are 0.73, 1.032 and 1.373, respectively. The data in Plates 2-4 indicate that for these flow conditions the initial standing wave has a "gradual" surface change rather than being a standing wave of "appreciable" height even with a maximum recommended wing-wall convergence ($|\Delta s/\Delta z| = 5$). In addition, even at the maximum attainable flow rates, $H_e/H_D = 1.67$, no well-defined or sudden change in water depth occurred.

c. Spreading of the initial standing wave across the chute. A small gravity wave tends to move across the chute according to[E1]

$$\beta \doteq \sin^{-1}\left(\frac{1}{F}\right) \tag{11}$$

in which β is the angle of the wave and $F = v/\sqrt{gd}$ is the local Froude number. A definition sketch and a plot of Equation 11 are given in Plate 6. The calculated variation of β, using the recorded depths along $z = 0.0$ in Equation 11, are shown in Figure 11. Data indicate that in the vicinity of the crest, the wave angle can be represented by a single function, $\beta = f(s/H_e)$, for the test chute. Using this function and integrating numerically along the chute

$$s_{i+1} = s_i + \Delta s \tag{12a}$$

$$z_{i+1} = z_i + \Delta s \tan\left(\frac{\beta_i + \beta_{i+1}}{2}\right) \tag{12b}$$

gives the spread of the wave for tests N1, N2, and N3 shown in Figure 12. The initial conditions (at i = 0) are:

$$s_o = 0.5H_e \tag{13a}$$

$$\beta_o = 85^o \tag{13b}$$

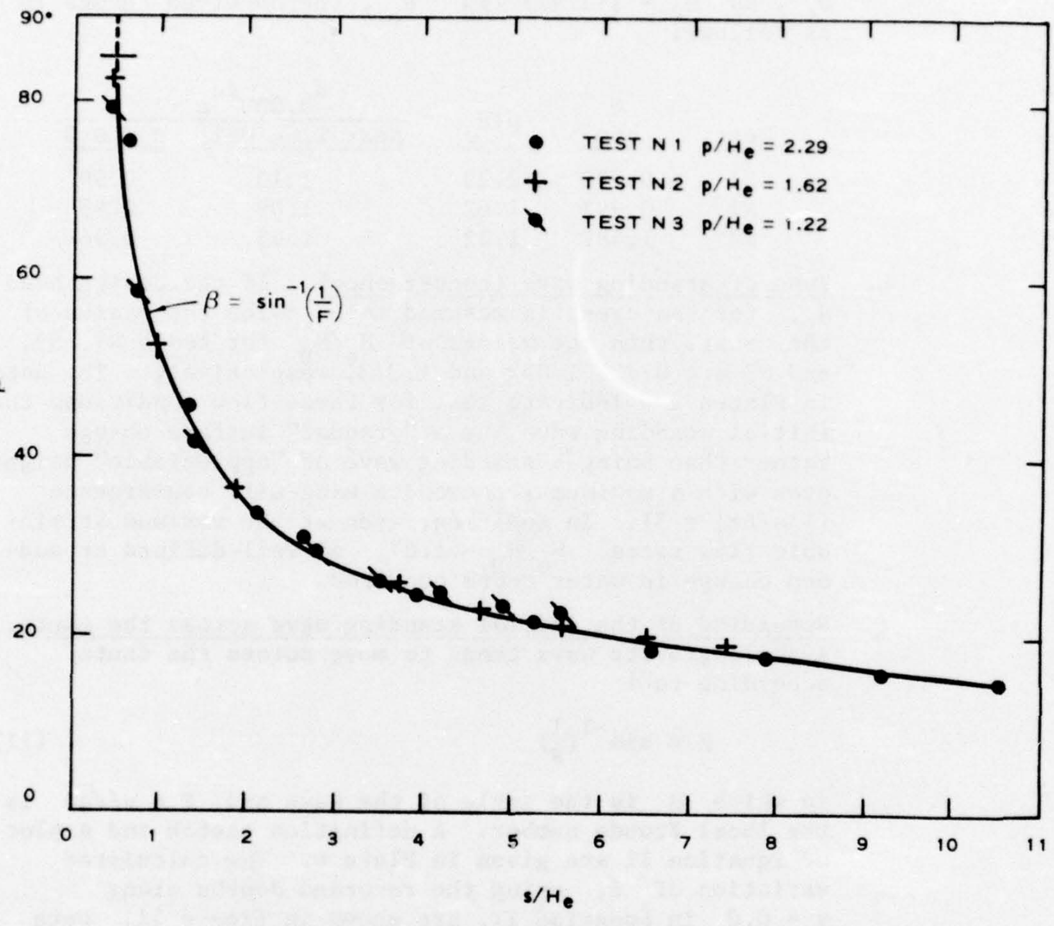


Figure 11. Orientation of a small gravity wave (z = 0.0)

$$z_o = 0.940 - s_o \tan 10.2^o \qquad (13c)$$

Locations of the computed wave are also shown in Plates 2, 3, and 4 and are in general agreement with the measured profiles.

d. Depths along the wall. For gradual wall deflections into the flow[E1]

$$\theta + \theta_1 = \sqrt{3}\ \tan^{-1}\sqrt{\frac{3}{F^2-1}} - \tan^{-1}\sqrt{\frac{1}{F^2-1}} \qquad (14)$$

in which θ_1 is an initial condition ($F = F_1$), and $\theta + \theta_1$ is the angle of deflection. Equation 14, with $1 \le F \le 4$ is also plotted in Plate 6; the value of $\theta + \theta_1$ varies from 0^o for an infinite Froude number to 65.88^o for $F = 1$. Data corresponding to Equation 14 are

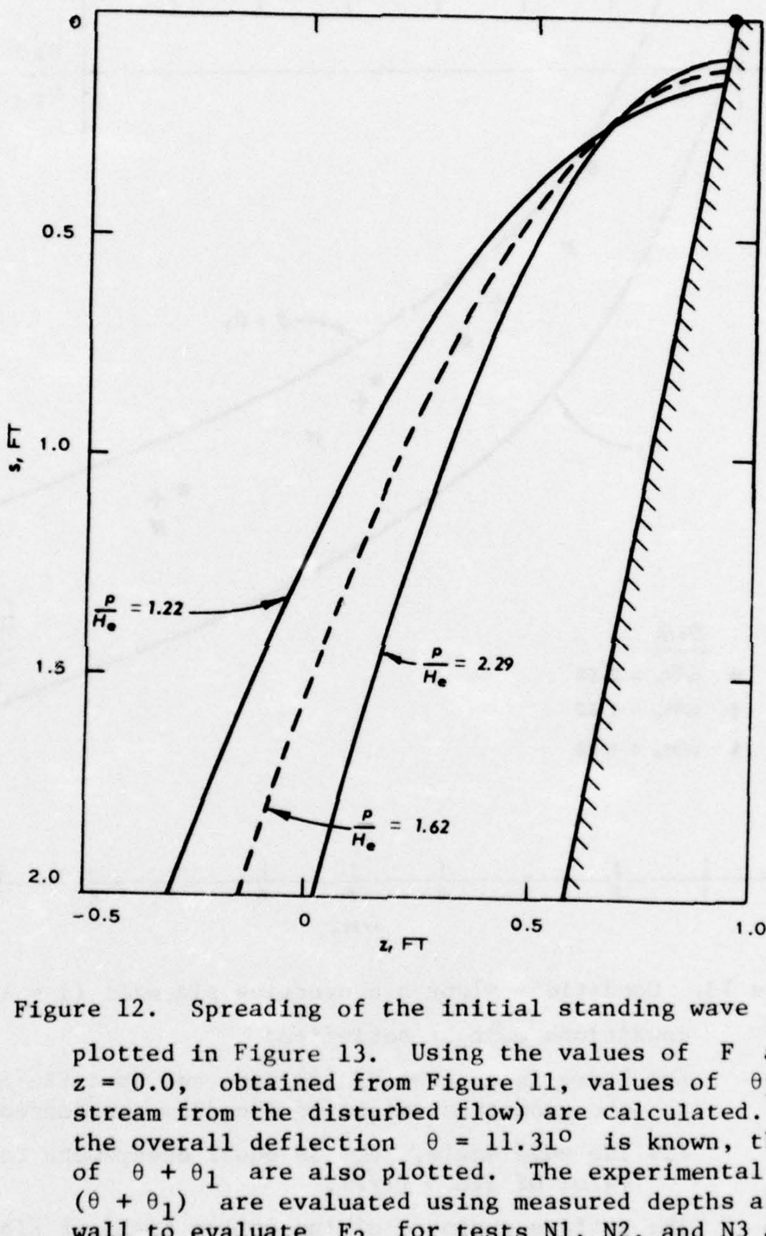


Figure 12. Spreading of the initial standing wave

plotted in Figure 13. Using the values of F along z = 0.0, obtained from Figure 11, values of θ_1 (upstream from the disturbed flow) are calculated. Since the overall deflection $\theta = 11.31^o$ is known, the values of $\theta + \theta_1$ are also plotted. The experimental data $(\theta + \theta_1)$ are evaluated using measured depths along the wall to evaluate F_2 for tests N1, N2, and N3 and are also shown in Figure 13. These data are in reasonable agreement with the values derived directly from Equation 14 using the mainstream F_1.

e. Surface contours in diverging flow. Channel expansions are analyzed by Ippen[E1]; in general, in expanding flow no steep wave fronts exist and the two following

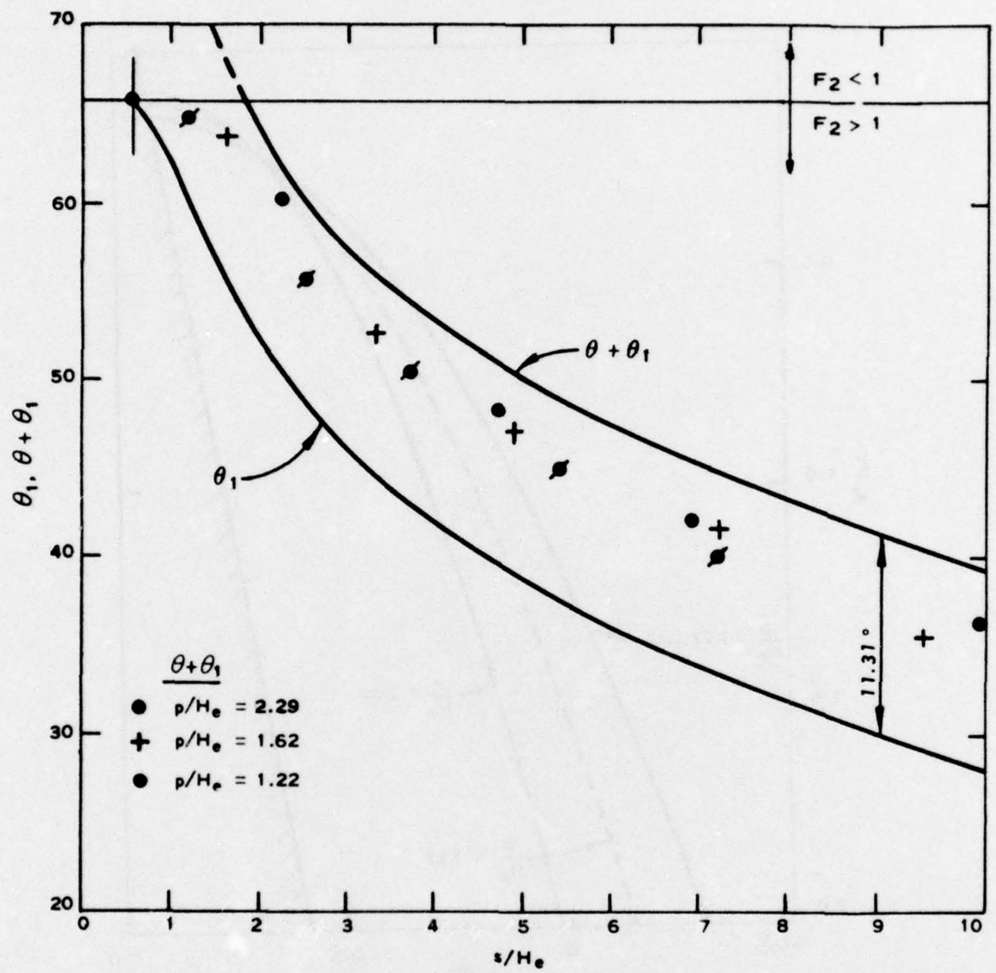


Figure 13. Conditions along a converging sidewall (θ = 11.31°)

conditions must be satisfied:

(1) There is an essentially constant specific head for the geometric extent of the flow considered.

(2) The wave angle, β, is equal everywhere to the local value of $\sin^{-1}$ (1/F).

The surface contours of the entire field of flow are then mapped via the method of characteristics. Rouse, Bhoota, and Hsu[E3] conducted experiments on channel expansions and found that the surface contours were adequately described by a simplified empirical function (Equation 3 in paragraph 13). Their experimentally determined surface contours for a 90° expansion on a horizontal floor with F_1

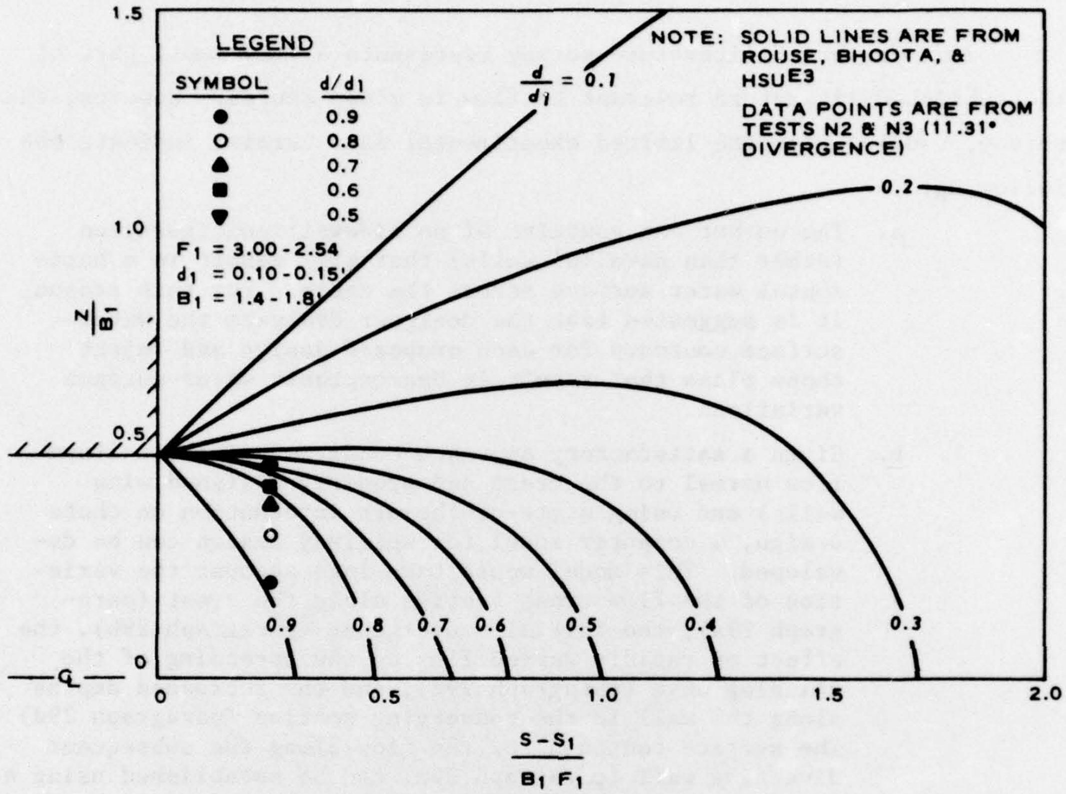


Figure 14. Divergence (surface contours)

equals 2.0 are approximately as shown in Figure 14. Five comparable data values for the 11.31° expansion (Tests N2 and N3 are also shown in Figure 14. For these tests, the value of B_1 is assumed equal to the width of the standing wave at the end of the convergence as obtained from Figure 12. In general, Rouse, Bhoota, and Hsu[E3] have previously established a reasonably complete data set for expansions in supercritical flow and since they have verified the accuracy of the method of characteristics solution for expansions, this aspect of the problem is not pursued further herein. The current data tend to indicate that the previous work can be satisfactorily applied to supercritical flow in a steep chute having convex sidewalls provided the divergence is not in a highly nonuniform flow region.

PART IV: CONCLUSIONS AND RECOMMENDATIONS

30. The brief literature survey represents a very small part of the published literature relevant to flow in steep chutes. However, the survey, coupled with the limited experimental data herein, indicate the following:

a. The author can conceive of no sidewall configuration (other than parallel walls) that will result in a horizontal water surface across the chute. For this reason, it is suggested that the designer evaluate the water-surface contours for each proposed design and reject those plans that result in unacceptable water-surface variations.

b. Given a satisfactory approach condition (i.e., uniform flow normal to the crest and properly designed wing walls) and using state-of-the-art information on chute design, a computer model for spillway design can be developed. This model would take into account the variation of the flow cross section along the crest (paragraph 29a), the initial convergence (paragraph 29b), the effect of rapidly varied flow on the spreading of the standing wave (paragraph 29c), and the increased depths along the wall in the converging section (paragraph 29d). The surface contours for the flow along the subsequent diverging wall (paragraph 29e) can be established using a programmed method-of-characteristics solution. A model that neglects any of the above factors cannot provide a solution that is entirely satisfactory to the designer.

c. Given an unsatisfactory approach condition it is unlikely that a computer model can provide a reliable sidewall design. For this particular circumstance, the hydraulic model is probably the best available design tool.

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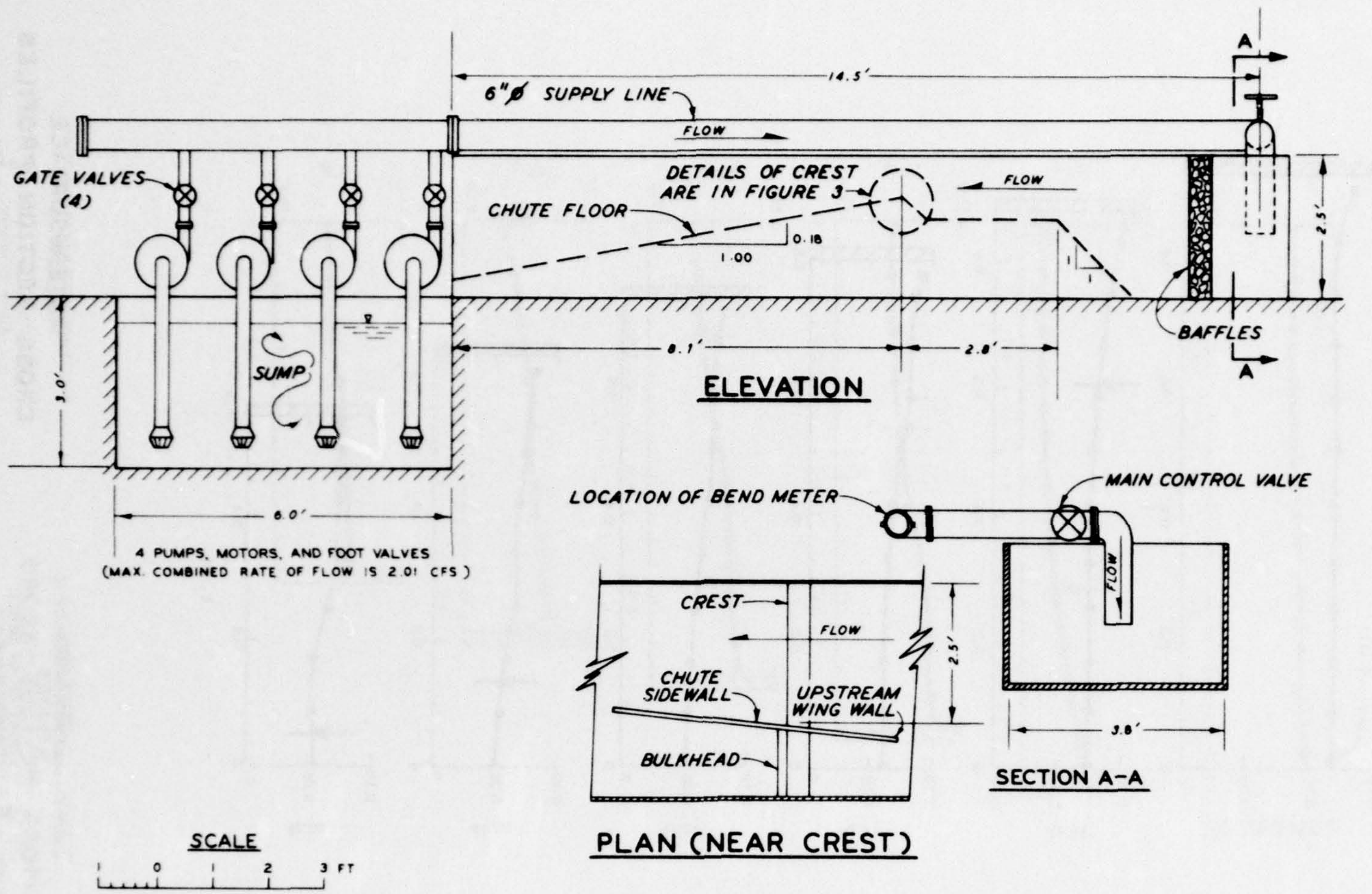

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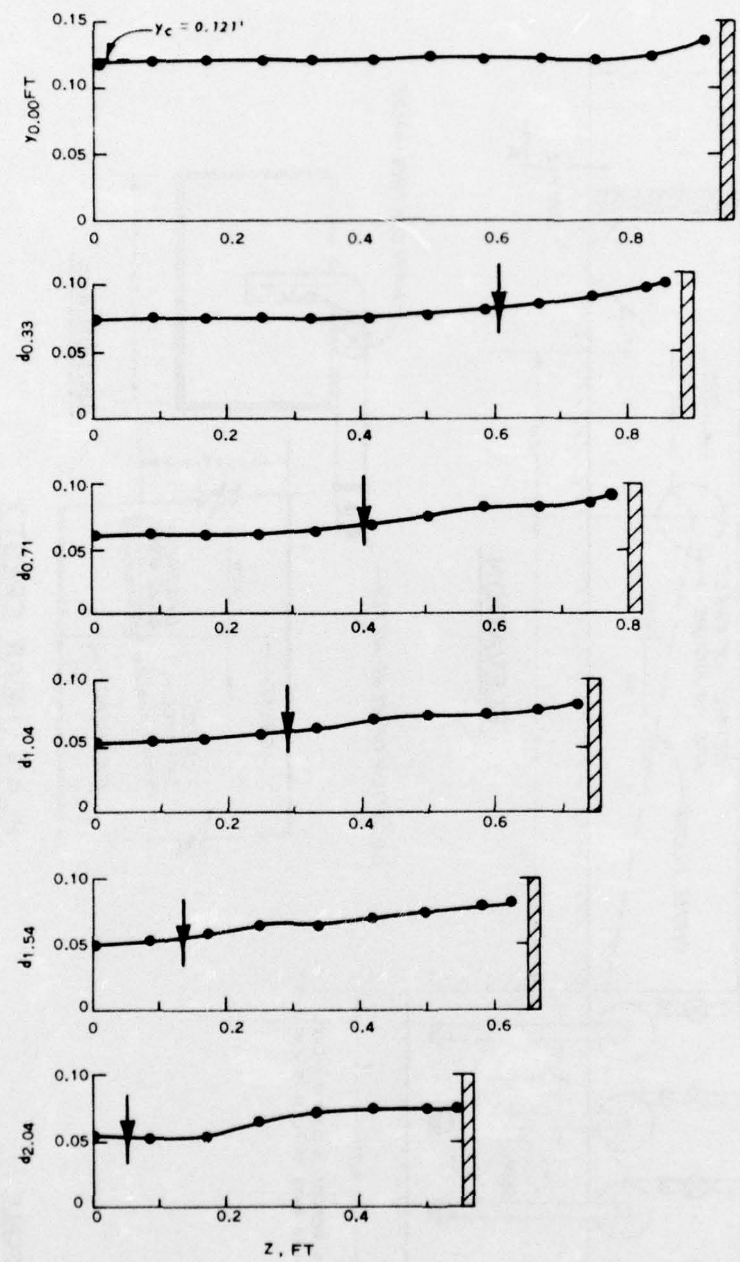
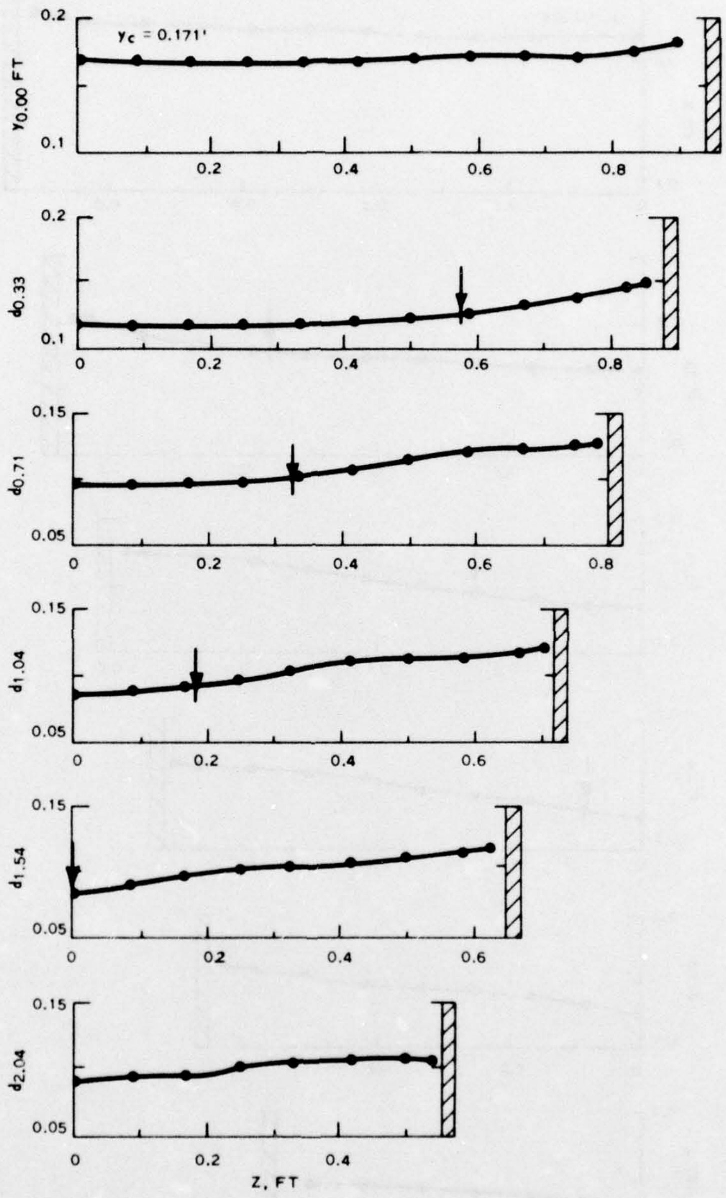


NOTE: DEFINITION SKETCH IS IN PLATE 5

↓ = LOCATION OF SMALL GRAVITY WAVE

WATER–SURFACE CROSS–SECTION PROFILES TEST N1, Q=0.59 CFS

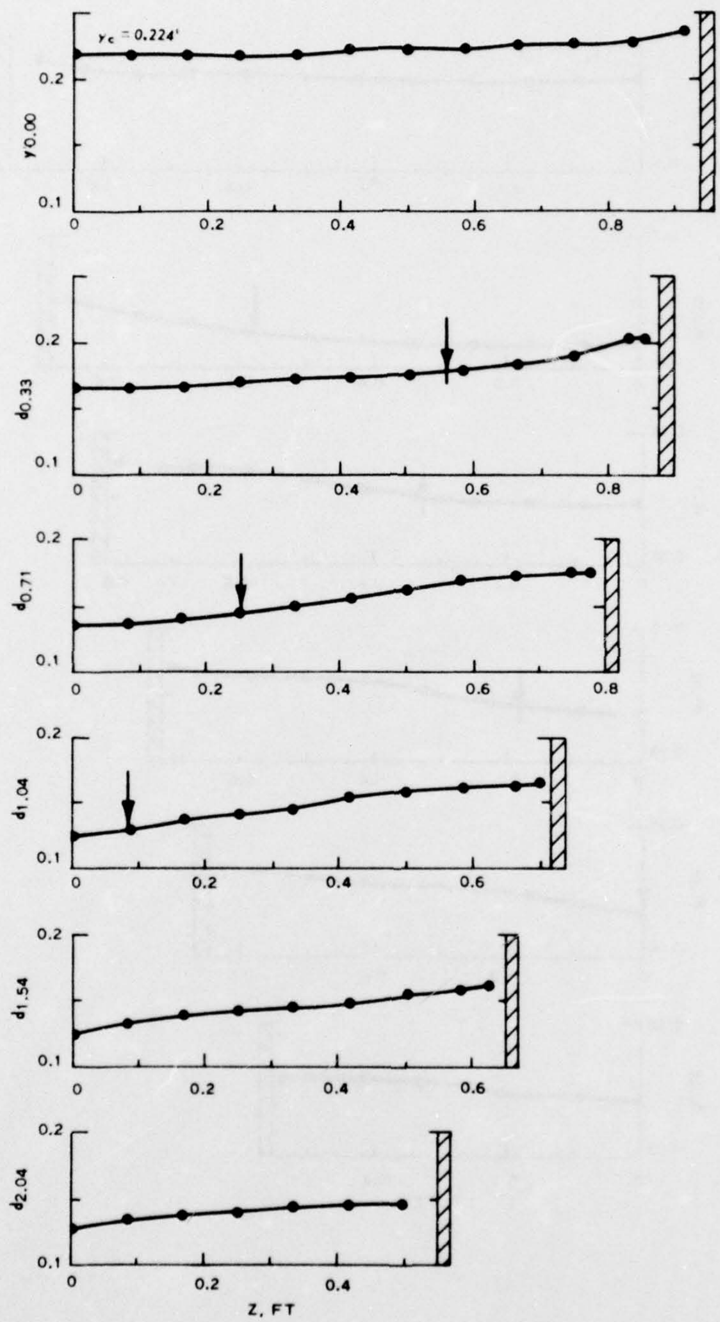
PLATE 2



NOTE: DEFINITION SKETCH IS IN PLATE 5

↓ = LOCATION OF SMALL GRAVITY WAVE

WATER-SURFACE CROSS-SECTION PROFILES TEST N2, Q=0.99 CFS

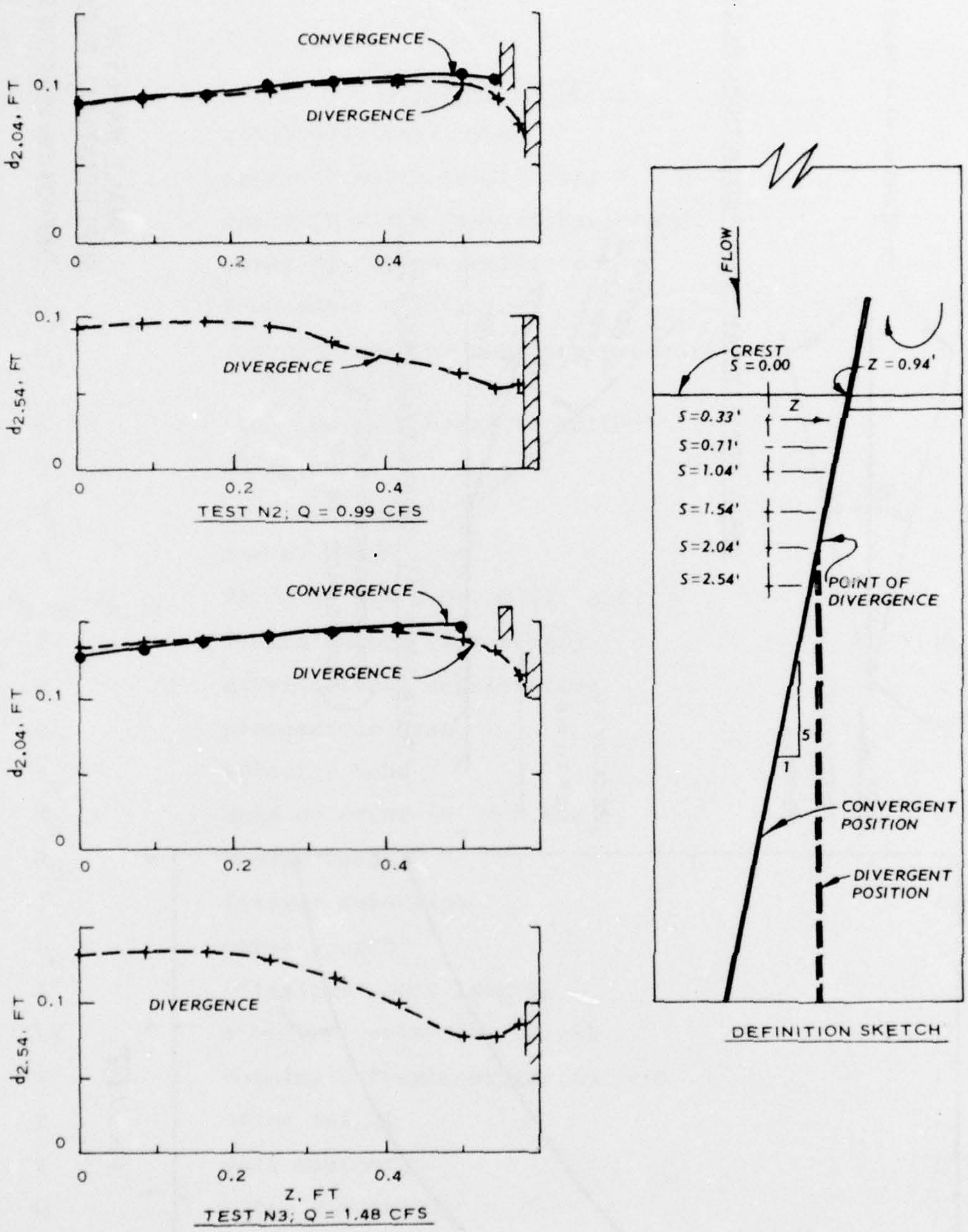


NOTE: DEFINITION SKETCH IS IN PLATE 5

↓ = LOCATION OF SMALL GRAVITY WAVE

WATER-SURFACE CROSS-SECTION PROFILES TEST N3, Q= 1.48 CFS

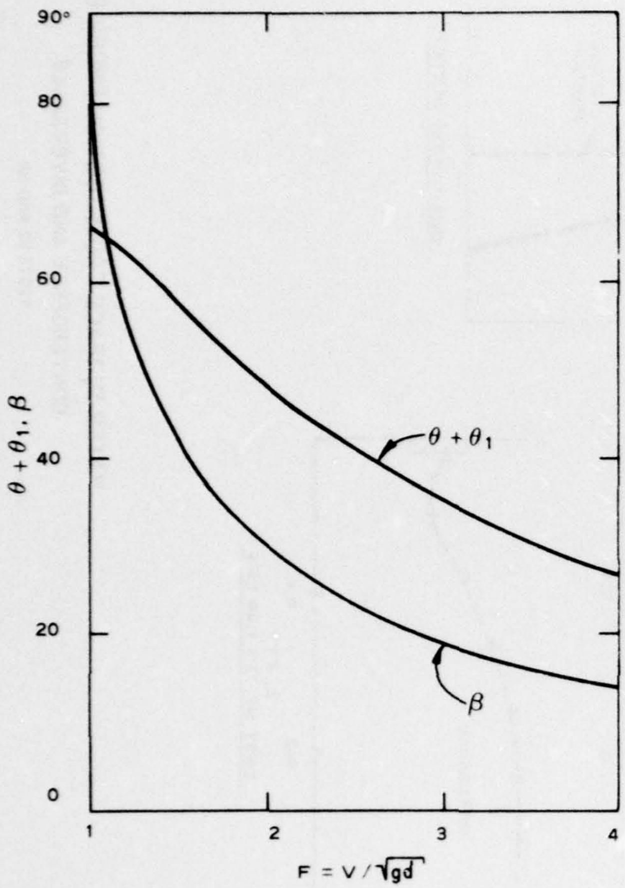
PLATE 4

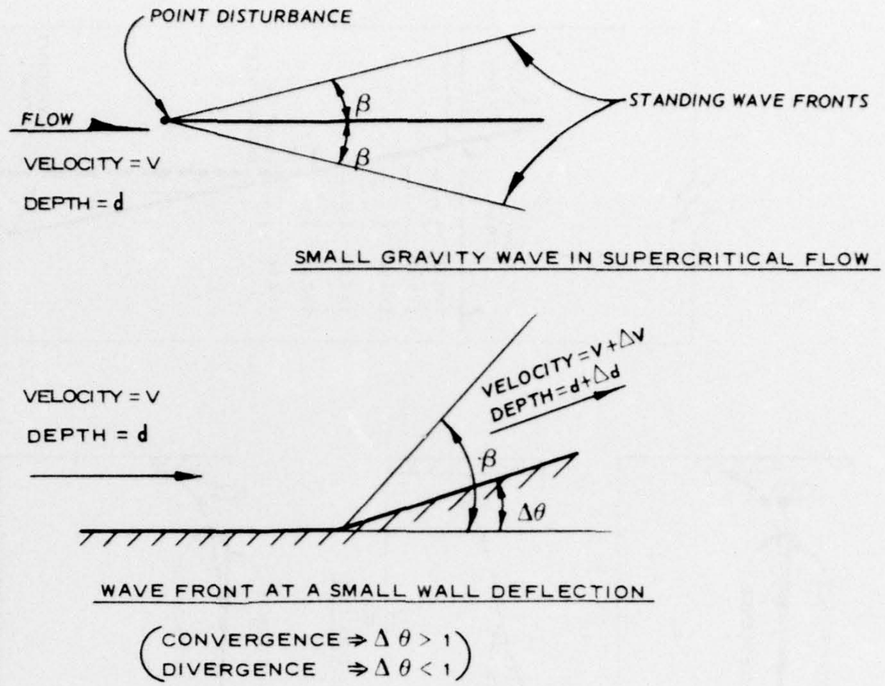


WATER-SURFACE CROSS-SECTION PROFILES

CONVERGENCE AND DIVERGENCE

TESTS N2 AND N3





SMALL WAVES IN SUPERCRITICAL FLOW

DEFINITION SKETCHES

APPENDIX A: NOTATION

SYMBOL	DESCRIPTION	DIMENSION (F-L-T)
a	cross-sectional area	L^2
B	approach width (supercritical flow)	L
B_a	approach width (subcritical flow)	L
C	crest discharge coefficient	$L^{1/2}T^{-1}$
C_M	bend meter coefficient	$L^{5/2}T^{-1}$
C_p	specific heat at constant pressure ($Btu/lb_m{}^{\circ}R$)	-
d	local depth (normal to bottom)	L
D	diameter	L
d_c	critical depth	L
d_n	normal depth	L
$d_{0.00}, d_{0.71}$,	depth at sta 0.00, 0.71, etc.	L
F	Froude number ($F = v/\sqrt{gd}$)	none
g	gravitational acceleration	LT^{-2}
h	piezometric head	L
h_v	velocity head	L
H_e	head on crest ($H_e = h + h_v$)	L
H_D	design head	L
i	integer subscript	none
L_c	crest length	L
L'	effective crest length	L
Lw	wing-wall extension length	L
n	Manning's roughness coefficient	$L^{-1/3}T$
p	crest height	L
q	unit discharge	L^2T^{-1}
Q	total discharge	L^3T^{-1}
r	radius	L
R	manometer differential	L
R_e	Reynolds number ($R_e = \frac{VD}{\nu}$)	none
s	dimension (measured parallel to channel bottom)	L

SYMBOL	DESCRIPTION	DIMENSION (F-L-T)
S	slope	none
T	temperature (°R)	-
u_x, u_y	velocity components	-
v	local velocity	LT^{-1}
V	mean velocity	LT^{-1}
x	dimension (horizontal)	L
y	dimension (vertical)	L
z	dimension (transverse)	L
β	deflection angle (wave)	none
θ	deflection angle (wall)	none
γ	ratio of specific heats	none
ν	kinematic viscosity	L^2T^{-1}
ρ	density	$FL^{-4}T^2$
s_f	slope of the energy grade line	none
s_o	slope of the channel bottom	none

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